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Motion of colored particle in a chromomagnetic field

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Abstract. The Dirac equation in a chromomagnetic field is solved for a colored particle moving in a limited space volume. Quantized energy levels and the corresponding wave functions are found for backgrounds both directed along the third axes and having spherical symmetry. It was shown that there exists an interrelation with the case of motion in an infinite space volume.

Introduction

Quantum and classical mechanics of non-abelian charged particles have been studied by various authors. Problems in these theories serve to find the solution of QCD problems and are connected with the description of the quark's motion inside hadrons or any other limited space volume. Since QCD has the confinement property, presumably it will be better to consider QCD problems in a finite volume as well [1]. As is known, there are color background fields in the QCD vacuum [2] and so it makes sense to study the problem of motion of a colored particle in a color background in limited space volume. The simplest case, a constant color background, could be given by two different types of vector potentials [3, 4]. Problems in an external color field given by abelian vector potentials are solved analogously to the ones in abelian theory. The second type of vector potentials are non-commuting constant vector potentials, which are not gauge equivalent to the first type [4]. These potentials are used to solve the ground state problem of QCD [4, 5] and other different problems connected with the QCD vacuum [5–9] as well and sometimes give physically different results from the ones in which first type potentials was used. Here we shall solve the Dirac equation in a finite space volume in a constant background chromomagnetic field given by the second type of vector potentials. The solutions and spectra found are more suited to the physical situation and by use of them could be constructed the quark's Green's function by the help of an exact solution method [10], which could be used in solving QCD problems.

1 Constant homogeneous background

Let us define an external chromomagnetic field by constant vector potentials. Within the SU(3) color symmetry group they look like

$$A_1^a = \sqrt{\tau}\delta_{1a},\ A_2^a = \sqrt{\tau}\delta_{2a},\ A_3^a = 0,\ A_0^a = 0, \qquad (1)$$

where $a = \overline{1,8}$ is a color index, τ is a constant and $\delta_{\mu a}$ is the Kronecker symbol. The field (1) is directed along the third axes of ordinary and color spaces:

$$F_{12}^3 = H_z^3 = g\tau, \quad \text{the other } F_{\mu\nu}^a = 0. \qquad (2)$$

Here g is the color interaction constant.

The Dirac equation for a colored particle in the external color field has the form

$$(\gamma^\mu P_\mu - M)\,\psi = 0, \qquad (3)$$

where $P_\mu = p_\mu + gA_\mu = p_\mu + gA_\mu^a \frac{\lambda^a}{2}$; the λ^a are the Gell-Mann matrices describing the particle's color spin. Equation (3) written for the Majorana spinors ϕ and χ has the well-known form

$$\left(\sigma^i P_i\right)^2 \psi = -\left(\frac{\partial^2}{\partial t^2} + M^2\right)\psi, \qquad (4)$$

where the Pauli matrices σ^i describe a particle's spin. Here and afterwards ψ means ϕ or χ. The spinors ϕ and χ have two components, corresponding to the two spin states of a particle

$$\psi = \begin{pmatrix}\psi_+ \\ \psi_-\end{pmatrix}.$$

Each component of ψ transforms under the fundamental representation of the color group SU(3) and has three color components describing the color states of a particle corresponding to three eigenvalues of the color spin λ^3:

$$\psi_\pm = \begin{pmatrix}\psi_\pm(\lambda^3=+1) \\ \psi_\pm(\lambda^3=-1) \\ \psi_\pm(\lambda^3=0)\end{pmatrix} = \begin{pmatrix}\psi_\pm^{(1)} \\ \psi_\pm^{(2)} \\ \psi_\pm^{(3)}\end{pmatrix}. \qquad (5)$$

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Since the background field (1) is time independent, the equation (4) has got the form

$$\left[\mathbf{p}^2+\frac{g^2\tau}{2}I_2+g\tau^{\frac{1}{2}}\left(p_1\lambda^1+p_2\lambda^2\mp\frac{H_z^3}{2}\lambda^3\right)\right]\psi_\pm = \left(E^2-M^2\right)\psi_\pm . \qquad (6)$$

Here

$$I_2=\begin{pmatrix}1&0&0\\0&1&0\\0&0&0\end{pmatrix}$$

is the color matrix. Writing this equation for the color components (5) we get two independent systems of differential equations:

$$\begin{cases}\mathbf{p}^2\psi_+^{(1)}+g\tau^{\frac{1}{2}}\left(p_1-\mathrm{i}p_2\right)\psi_+^{(2)}=\left(E^2-M^2\right)\psi_+^{(1)},\\ \left(\mathbf{p}^2+g^2\tau\right)\psi_+^{(2)}+g\tau^{\frac{1}{2}}\left(p_1+\mathrm{i}p_2\right)\psi_+^{(1)}=\left(E^2-M^2\right)\psi_+^{(2)},\\ \mathbf{p}^2\psi_+^{(3)}=\left(E^2-M^2\right)\psi_+^{(3)},\end{cases}$$

$$\begin{cases}\left(\mathbf{p}^2+g^2\tau\right)\psi_-^{(1)}+g\tau^{\frac{1}{2}}\left(p_1-\mathrm{i}p_2\right)\psi_-^{(2)}=\left(E^2-M^2\right)\psi_-^{(1)},\\ \mathbf{p}^2\psi_-^{(2)}+g\tau^{\frac{1}{2}}\left(p_1+\mathrm{i}p_2\right)\psi_-^{(1)}=\left(E^2-M^2\right)\psi_-^{(2)},\\ \mathbf{p}^2\psi_-^{(3)}=\left(E^2-M^2\right)\psi_-^{(3)}.\end{cases} \qquad (7)$$

From (7) we get the same equation for all states $\psi_\pm^{(i)}$ $(i=1,2)$:

$$\left[\left(\mathbf{p}^2+\frac{g^2\tau}{2}+M^2-E^2\right)^2-g^2\tau\left(p_\perp^2+\frac{g^2\tau}{4}\right)\right]\psi_\pm^{(i)}=0, \qquad (8)$$

where $p_\perp^2=p_1^2+p_2^2$. This equation possesses rotational invariance around the z axis. If we consider free motion of the particle in all $\mathbf{x}$ space, then we should not impose any boundary condition on the solution of (8) and shall get continuous energy spectra as found in [11,12] for plane wave solutions $\psi_\pm^{(i)}\sim \mathrm{e}^{\mathrm{i}\mathbf{pr}}$,

$$E_{1,2}^2=\left(\sqrt{p_\perp^2+\frac{g^2\tau}{4}}\mp\frac{g\tau^{\frac{1}{2}}}{2}\right)^2+p_3^2+M^2. \qquad (9)$$

This spectrum is used in solving various problems [5–9]. Since free motion of the colored particle in nature could take place in limited space, for example inside a hadron, let us consider motion of the particle in the limited space bounded by a cylinder with radius r_0 and height z_0 and solve (8) with the boundary conditions $\psi_\pm^{(i)}(r=r_0,z)=0$, $\psi_\pm^{(i)}(r,z_0)=0$ $\left(r^2=x^2+y^2\right)$. Let us rewrite (8) in the form

$$\left[\left(-\nabla^2+\frac{g^2\tau}{4}\right)^2-2\left(E^2-M^2+\frac{g^2\tau}{4}\right)\left(-\nabla^2+\frac{g^2\tau}{4}\right)+\left(E^2-M^2-\frac{g^2\tau}{4}\right)^2-g^2\tau\frac{\partial^2}{\partial z^2}\right]\psi_\pm^{(i)}=0. \qquad (10)$$

With $\left(-\nabla^2+\frac{g^2\tau}{4}\right)\psi_\pm^{(i)}=\eta_\pm^{(i)}$ it has got the following form:

$$\left(-\nabla^2+\frac{g^2\tau}{4}\right)\eta_\pm^{(i)}-2\left(E^2-M^2+\frac{g^2\tau}{4}\right)\eta_\pm^{(i)}+\left(\left(E^2-M^2-\frac{g^2\tau}{4}\right)^2-g^2\tau\frac{\partial^2}{\partial z^2}\right)\psi_\pm^{(i)}=0. \qquad (11)$$

Acting on equation (11) by the operator $\left(-\nabla^2+\frac{g^2\tau}{4}\right)$ we get an equation for $\eta_\pm^{(i)}$ that has the same form as for $\psi_\pm^{(i)}$, i.e., (10) with the replacement $\psi_\pm^{(i)}$ on $\eta_\pm^{(i)}$. This means that $\eta_\pm^{(i)}$ and $\psi_\pm^{(i)}$ differs only by a general constant multiplier $k^2:\eta_\pm^{(i)}=k^2\psi_\pm^{(i)}$ or

$$\left(-\nabla^2+\frac{g^2\tau}{4}\right)\psi_\pm^{(i)}=k^2\psi_\pm^{(i)}. \qquad (12)$$

Equation (12) is equivalent to (10). In the appendix is shown another way to reduce (10) to the form (12). In a cylindrical coordinate system the solution of (12) can be looked for using the separation ansatz $\psi_\pm^{(i)}(\mathbf{r})=\psi(r)\cdot u(\varphi)\cdot v(z)$, and (12) is divided into three independent equations:

$$\begin{cases}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi(r)}{\partial r}\right)+\left(k^2-\frac{g^2\tau}{4}-\lambda^2-\frac{m^2}{r^2}\right)\psi(r)=0,\\ -\frac{\partial^2u(\varphi)}{\partial\varphi^2}=m^2u(\varphi),\\ \frac{\partial^2v(z)}{\partial z^2}+\lambda^2v(z)=0.\end{cases} \qquad (13)$$

The solutions of the last two equations with the boundary condition $v(z_0)=0$ are $u(\varphi)=\frac{1}{\sqrt{2\pi}}\mathrm{e}^{\mathrm{i}m\varphi}$ $(m=0,\pm1,\pm2,\ldots)$ and $v(z)=\sin p_3z(\lambda^2=p_3^2,p_3=\frac{\pi n}{z_0})$. For the $p_3=0$ case we can choose $v(z)=1$. It is worth to remark that (12) and (10) become equivalent after taking the last equation in them into account, and there is no such problem for the case $p_3=0$. Taking into account the explicit expression of $v(z)$ and (12) in (10) we get the following equation for the constant k^2:

$$\left(k^2\right)^2-2k^2\left(E^2-M^2+\frac{g^2\tau}{4}\right)+\left(E^2-M^2-\frac{g^2\tau}{4}\right)^2+g^2\tau p_3^2=0,$$

from which one finds the following expression for the constant k^2:

$$k_{1,2}^2=\left(\sqrt{E^2-M^2-p_3^2}\pm\frac{g\tau^{\frac{1}{2}}}{2}\right)^2+p_3^2. \qquad (14)$$

The first equation in (13) is the Bessel equation. Under the finiteness condition of $r\to 0$ it has a solution determined by the Bessel function,

$$\psi_{1,2}(r) = J_m\left(r\sqrt{k_{1,2}^2 - p_3^2 - \frac{g^2\tau}{4}}\right). \tag{15}$$

If we do not impose any boundary condition on the states (15) with given m, then they have an energy from the continuous spectrum (9), which does not depend on m. Imposing the boundary condition $\psi_{1,2}(r_0) = 0$ we find two branches of quantized energy levels of the colored particle in a chromomagnetic field (1):

$$\left(E_m^{(N)}\right)_{1,2}^2 = \left(\sqrt{\frac{\left(\alpha_m^{(N)}\right)^2}{r_0^2} + \frac{g^2\tau}{4}} \mp \frac{g\tau^{\frac{1}{2}}}{2}\right)^2 + p_3^2 + M^2. \tag{16}$$

Here $\alpha_m^{(N)}$ are the zeros of the Bessel function, and the quantity N labels the sequence of zeros, where $N = 1, 2, 3, \ldots$. From (16) one sees that these energy branches, as branches of the continuous spectrum (9), are defined by the $\pm$ sign of the field strength, which is not connected with the direction of the field. The energy levels $E_m^{(N)}$ are determined by the m chromomagnetic quantum number, i.e. by the projection of the chromomagmetic moment of the particle onto the chromomagnetic field. Another quantum number is N, determining the energy level $E_m^{(N)}$, a result of quantization due to the finiteness of the motion. In contrast to the discrete spectum (16) the continuous spectum (9) is not determined by the quantum number m. Thus, finiteness of the space volume turns the continuous spectra (9) for a given m into a discrete series determined by the quantum number N. Both spectra (9) and (16) are infinitely degenerate. Since (12) is universal for the spin $\pm$ and color (i) indices, any of these states may have any energy from the spectra (16) or (9). If in the energy branches (16) one makes the replacement $\frac{\alpha_m^{(N)}}{r_0} = p_\perp$ we get the continuous spectra (9). This is suitable to quantization of momentum in standing waves. Taking into account the spectrum (16) in (14) we find the value of constant k^2,

$$k_{1,2}^2 = \left(k_m^{(N)}\right)^2 = \frac{\left(\alpha_m^{(N)}\right)^2}{r_0^2} + \frac{g^2\tau}{4} + p_3^2,$$

which is the same for both energy branches and so, the solution (15) is the same for these branches. Thus, the wave functions $\psi_\pm^{(i)}(\mathbf{r})$ from the chosen energy branch $\left(E_m^{(N)}\right)_1$ or $\left(E_m^{(N)}\right)_2$ are equal to

$$\psi_\pm^{(i)}(\mathbf{r}) = \sum_{m=-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}} \mathrm{e}^{im\varphi} \sin(p_3 z)\, J_m\left(r\frac{\alpha_m^{(N)}}{r_0}\right). \tag{17}$$

Thus, the motion of the colored particle in the field (1) along the z axis is a standing wave and in the (x, y) plane is a rotation. Writing (13) for the radius a of the circle in which particle rotates:

$$\frac{1}{a}\frac{\partial}{\partial r}\left(a\frac{\partial\psi(a)}{\partial r}\right) + \left(k^2 - \frac{g^2\tau}{4} - p_3^2 - \frac{m^2}{a^2}\right)\psi(a) = 0,$$

we can find discrete values for these orbits:

$$a_m^{(N)} = \frac{m}{\alpha_m^{(N)}} r_0.$$

In the continuous spectrum case the constant k^2 and radius a are

$$k^2 = \mathbf{p}^2 + \frac{g^2\tau}{4},\ a = \frac{m}{p_\perp},$$

and the solution (15) becomes

$$\psi_{1,2}(r) = \psi(r) = J_m(rp_\perp).$$

The states $\psi_\pm^{(3)}(\mathbf{r})$ correspond to states of a freely moving colorless scalar particle.

2 Spherically symmetric configuration of background

Let us consider the case of spherical symmetry in the ordinary space with constant chromomagmetic field defined by the non-commuting potentials

$$A_1^a = \sqrt{\tau}\delta_{1a},\ A_2^a = \sqrt{\tau}\delta_{2a},\ A_3^a = \sqrt{\tau}\delta_{3a},\ A_0^a = 0. \tag{18}$$

The non-zero components of the field strength tensor are

$$F_{23}^1 = F_{31}^2 = F_{12}^3 = g\tau.$$

In the external field (18) the explicit form of (4) is

$$\left(\mathbf{p}^2 + M^2 + \frac{3g^2\tau}{4} + g\tau^{\frac{1}{2}}\lambda^a p^a - \frac{g^2\tau}{2}\sigma^a\lambda^a\right)\Psi = E^2\Psi. \tag{19}$$

Let us denote the components of the spinor Ψ by ψ_1 and ψ_2:

$$\Psi = \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}.$$

Since the external field has three components in both spaces, in this case, different from the previous one, the components $\psi_{1,2}^{(a)}$ do not describe states with a definite value of he spin and color spin projection onto the field. Equation (19) can be written in terms of a system of differential equations for the components $\psi_{1,2}^{(a)}$

$$\begin{cases}\left(A + g\tau^{\frac{1}{2}}p_3 - \frac{g^2\tau}{2}\right)\psi_1^{(1)} + g\tau^{\frac{1}{2}}(p_1 - \mathrm{i}p_2)\psi_1^{(2)} = 0,\\ \left(A - g\tau^{\frac{1}{2}}p_3 + \frac{g^2\tau}{2}\right)\psi_1^{(2)} + g\tau^{\frac{1}{2}}(p_1 + \mathrm{i}p_2)\psi_1^{(1)} = g^2\tau\psi_2^{(1)},\\ (\mathbf{p}^2 + M^2)\psi_1^{(3)} = E^2\psi_1^{(3)},\\ \left(A + g\tau^{\frac{1}{2}}p_3 + \frac{g^2\tau}{2}\right)\psi_2^{(1)} + g\tau^{\frac{1}{2}}(p_1 - \mathrm{i}p_2)\psi_2^{(2)} = g^2\tau\psi_1^{(2)},\\ \left(A - g\tau^{\frac{1}{2}}p_3 - \frac{g^2\tau}{2}\right)\psi_2^{(2)} + g\tau^{\frac{1}{2}}(p_1 + \mathrm{i}p_2)\psi_2^{(1)} = 0,\\ (\mathbf{p}^2 + M^2)\psi_2^{(3)} = E^2\psi_2^{(3)},\end{cases} \tag{20}$$

where the operator A denotes $A = \mathbf{p}^2 + M^2 + \frac{3g^2\tau}{4} - E^2$. From the system (20) we get the same differential equation for all states $\psi_{1,2}^{(i)}$ $(i = 1, 2)$:

$$\left[\left(A - \frac{g^2\tau}{2}\right)^2 - g^2\tau\mathbf{p}^2\right]\left[\left(A + \frac{g^2\tau}{2}\right)^2 - g^2\tau\left(\mathbf{p}^2 + g^2\tau\right)\right] \times\psi_{1,2}^{(i)} = 0, \quad (21)$$

which possesses rotational invariance. Since the operator in the first square brackets commutes with the second one, (21) could be divided into two equations:

$$\left[\left(A - \frac{g^2\tau}{2}\right)^2 - g^2\tau\mathbf{p}^2\right]\psi_{1,2}^{(i)} = 0,$$
$$\left[\left(A + \frac{g^2\tau}{2}\right)^2 - g^2\tau\left(\mathbf{p}^2 + g^2\tau\right)\right]\psi_{1,2}^{(i)} = 0. \quad (22)$$

From (22) we get four branches of the continuous energy spectrum found in [11] for the plane wave solutions $\psi_{1,2}^{(i)} \sim \mathrm{e}^{\mathrm{i}\mathbf{pr}}$:

$$E_{1,2}^2 = \left(\sqrt{\mathbf{p}^2} \mp \frac{g\tau^{\frac{1}{2}}}{2}\right)^2 + M^2,$$
$$E_{3,4}^2 = \left(\sqrt{\mathbf{p}^2 + g^2\tau} \mp \frac{g\tau^{\frac{1}{2}}}{2}\right)^2 + M^2. \quad (23)$$

In the same manner as used in the previous section from (22) we get the equivalent equations

$$\left(-\nabla^2\right)\psi_j^{(i)} = k_{1,2}^2\psi_j^{(i)}, \quad (24)$$
$$\left(-\nabla^2 + g^2\tau\right)\psi_j^{(i)} = k_{3,4}^2\psi_j^{(i)} \qquad (i, j = 1, 2).$$

If we take (24) into account corresponding equations (22) we find the values of k_d^2:

$$k_{1,2}^2 = k_{3,4}^2 = k^2 = \left(\sqrt{E^2 - M^2} \pm \frac{g\tau^{\frac{1}{2}}}{2}\right)^2. \quad (25)$$

The solution of (24) can be found in textbooks of quantum mechanics [13]. In a spherical coordinate system for the wave functions $\psi_j^{(i)}$ one used the separation ansatz, i.e. it is chosen as a multiplication of the radial $R(r)$ and angular $Y_l^m(\theta, \varphi)$ parts:

$$\psi_j^{(i)}(\mathbf{r}) = R(r) \cdot Y_l^m(\theta, \varphi).$$

Here $r = \sqrt{x^2 + y^2 + z^2}$, l and m are the orbital angular momentum and chromomagnetic quantum numbers; θ and φ are the polar and azimuthal angles, respectively. The spherical functions $Y_l^m(\theta, \varphi)$ are expressed by means of the Legendre polynomials $P_l^{|m|}(\cos\theta)$

$$Y_l^m(\theta, \varphi) = a_m\sqrt{\frac{(2l+1)}{4\pi}\frac{(l- \mid m \mid)!}{(l+ \mid m \mid)!}}P_l^{|m|}(\cos\theta)\,\mathrm{e}^{\mathrm{i}m\varphi}, \quad (26)$$

with

$$a_m = \begin{cases} 1 & \text{for } m \geq 0, \\ (-1)^m & m < 0. \end{cases}$$

The equations for the radial parts of the wave functions are the same as for many quantum mechanical problems possessing rotational invariance,

$$\nabla_r^2 R(r) + \left(k_\nu^2 - \frac{l\,(l+1)}{r^2}\right)R(r) = 0, \quad (27)$$

where $\nu = 1, 2, 3, 4$; $k_{1,2}^2 = k^2$, $k_{3,4}^2 = k^2 - g^2\tau$. With the notation $Q(r) = \sqrt{r}R(r)$, (27) turns into Bessel's equation for $Q(r)$:

$$Q''(r) + \frac{1}{r}Q'(r) + \left(k_\nu^2 - \frac{\left(l + \frac{1}{2}\right)^2}{r^2}\right)Q(r) = 0. \quad (28)$$

The function $R(r)$ must be finite on $r \to 0$. This means that the solutions of (28) are first type Bessel functions:

$$R_l^\nu = \frac{C_l^\nu}{k_\nu\sqrt{r}}J_{l+1/2}(k_\nu r). \quad (29)$$

Imposing the boundary conditions $R_l^\nu(r_0) = 0$ we enclose the motion of the particle by a sphere with a radius r_0, where r_0 agrees with the hadron's size. Since the external field (18) does not depend on r, we have got the same equation for $R(r)$ as for a freely moving particle enclosed in a sphere, differing only from the expression by k^2 constant. From these boundary conditions one finds the quantized energy levels of the spectrum branches,

$$\left(E_l^{(N)}\right)_{1,2}^2 = \left(\frac{\alpha_l^{(N)}}{r_0} \mp \frac{g\tau^{\frac{1}{2}}}{2}\right)^2 + M^2, \quad (30)$$
$$\left(E_l^{(N)}\right)_{3,4}^2 = \left(\sqrt{\left(\frac{\alpha_l^{(N)}}{r_0}\right)^2 + g^2\tau} \mp \frac{g\tau^{\frac{1}{2}}}{2}\right)^2 + M^2.$$

In this case the boundary condition on $R_l^\nu(r)$ with fixed value of the angular momentum quantum number l chooses from the continuous energy spectrum (23) the discrete series (30) of the radial quantum number N for this value of l, and the energy degeneracy for this state remains infinite. The replacement connecting discrete and continuous spectra is $\frac{\alpha_l^{(N)}}{r_0} = \mid \mathbf{p} \mid$. By means of the spectra (30) one finds the same values for k_ν:

$$k_\nu = k_l^{(N)} = \frac{\alpha_l^{(N)}}{r_0}.$$

The radius a of the particle could be found from (27) using the maximum condition $R'(a) = 0$ and is equal to

$$a_l^{(N)} = r_0\frac{\sqrt{l\,(l+1)}}{\alpha_l^{(N)}}.$$

As the energy spectrum is described by (30), the motion orbits of the particle are quantized and defined by the orbital and radial quantum numbers l and N as well. Thus, the general solution of (24) is

$$\psi_j^{(i)} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} C_l^m Y_l^m(\theta,\varphi) \frac{r_0}{\alpha_l^{(N)}\sqrt{r}} J_{l+1/2}(kr)\,. \quad (31)$$

In the case of this field configuration the particle moves on "s","p","d","f",... orbitals known from orbital angular momentum eigenfuctions. For the continuous spectrum the constant k and radius a are

$$k = |\mathbf{p}|\,,\ a_l(p) = \frac{\sqrt{l(l+1)}}{|\mathbf{p}|}.$$

The plane wave could be decomposed by means of spherical waves and therefore the plane wave solution obeys the initial equation as well [13].

Using the energy spectra (16) and (30), and estimates for the field strength of the gluon condensate [2] and of the hadron's size, the energy of a photon emitted by an excited hadron may be calculated. Using the wave functions (17) and (31) the explicit expressions for the Green's functions for these particles at finite temperature can be found [10], which gives a possibility to investigate different radiation corrections in the chromomagnetic field and the quark condensate,

$$\langle\overline{\psi}\psi\rangle = -\mathrm{i}\lim_{z\to 0}\mathrm{Tr}\left[G(z) - G(z)\,|_{A=0}\right]. \quad (32)$$

The spectra found, (16) and (30), and the quark condensate (32) open new possibilities to investigate chromomagnetic catalysis of color superconductivity as well [6].

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Appendix

It is easy to reduce (11) to the form (13) in the momentum representation:

$$\psi_\pm^{(i)}(\mathbf{r}) = \int_{-\infty}^{+\infty} \frac{\mathrm{d}^3p}{(2\pi)^{\frac{3}{2}}} \mathrm{e}^{\mathrm{i}\mathbf{pr}} \widetilde{\psi}_\pm^{(i)}(\mathbf{p})\,.$$

Then (11) will have the form

$$\left[\left(\mathbf{p}^2 + \frac{g^2\tau}{2} + M^2 - E^2\right)^2 - g^2\tau\left(p_\perp^2 + \frac{g^2\tau}{4}\right)\right] \times \widetilde{\psi}_\pm^{(i)}(\mathbf{p}) = 0, \quad (33)$$

which is divided into two equations

$$\left[\left(\mathbf{p}^2 + \frac{g^2\tau}{2} + M^2 - E^2\right) \pm g\tau^{1/2}\sqrt{p_\perp^2 + \frac{g^2\tau}{4}}\right] \times \widetilde{\psi}_\pm^{(i)}(\mathbf{p}) = 0,$$

and can be written in the following form:

$$\left(\sqrt{p_\perp^2 + \frac{g^2\tau}{4}} \pm \frac{g\tau^{\frac{1}{2}}}{2}\right)^2 \widetilde{\psi}_\pm^{(i)}(\mathbf{p}) = \left(E^2 - M^2 - p_3^2\right)\widetilde{\psi}_\pm^{(i)}(\mathbf{p})\,. \quad (34)$$

From (34) we get

$$\sqrt{p_\perp^2 + \frac{g^2\tau}{4}}\left(\widetilde{\psi}_\pm^{(i)}(\mathbf{p})\right)^{1/2} = \left(\sqrt{E^2 - M^2 - p_3^2} \mp \frac{g\tau^{\frac{1}{2}}}{2}\right)\left(\widetilde{\psi}_\pm^{(i)}(\mathbf{p})\right)^{1/2}.$$

Squaring the last equality and returning to the coordinate representation by an inverse Fourier transformation we get (12):

$$\left(-\nabla_{r,\varphi}^2 + \frac{g^2\tau}{4}\right)\psi_\pm^{(i)} = \left(\sqrt{E^2 - M^2 - p_3^2} \mp \frac{g\tau^{\frac{1}{2}}}{2}\right)^2 \psi_\pm^{(i)}.$$

References

1. P. van Baal, QCD in a Finite Volume, hep-ph/0008206
2. O. Nachtmann, High Energy Collisions and Nonperturbative QCD, in Lectures on QCD. Applications, edited by F.L.H. Griesshammer, D. Stoll (Springer 1997)
3. G.K. Savvidy, Phys. Lett. B **71**, 133 (1979)
4. L.S. Brown, W.I. Weisberger, Nucl. Phys. B **157**, 285 (1979); M. Reuter, C. Wetterich, Phys. Lett. B **334**, 412 (1994)
5. R. Parthasarathy, M. Singer, C. Viswanatham, Canad. J. Phys. **61**, 1442 (1983); W.I. Weisberger, Nucl. Phys. B **161**, 61 (1979); T.N. Tudron, Phys. Rev. D **22**, 2566 (1980)
6. D. Ebert, K.G. Klimenko, H. Toki, V.Ch. Zhukovsky, Prog. Theor. Phys. **106**, 835 (2001); D. Ebert, K.G. Klimenko, H. Toki, Phys. Rev. D **64**, 014038 (2001); K.G. Klimenko, A.S. Vshivtsev, B.V. Magnitsky, Nuovo Cimento, A **107**, 439 (1994); I.A. Shovkovy, V.M. Turkowsky, Phys. Lett. B **367**, 213 (1996)
7. Sh. Mamedov, Eur. Phys. J. C **18**, 523 (2001); Sh. Mamedov, Jian-zu Zhang, V.Ch. Zhukovskii, Mod. Phys. Lett. A, **16**, 845 (2001); V.Ch. Zhukovskii, Sh. Mamedov, Izv. Vissh. Uch. Zaved. Fizika N**10**, 23 (1990); A.V. Averin, A.V. Borisov, V.Ch. Zhukovskii, Z. Phys. C **48**, 457 (1990); A. Mil'shtein, Yu. Pinelis, Z. Phys. C **27**, 461 (1985); D. Ebert, V.Ch. Zhukovsky, Mod. Phys. Lett. A **12**, 2567 (1997)
8. A. Cabo, A. Shabad, Gauge Fields in the Medium and Vacuum in Presence of External Potential, Proceedings Lebedev Phys. Inst. **192**, 153 (1988); Sh.S. Agaev, A.S. Vshivtsev, V.Ch. Zhukovsky, Yad. Fiz. **36**, 1023 (1992); Sh.S. Agaev, A.S. Vshivtsev, V.Ch. Zhukovsky, O.F. Semyonov, Sov. Phys. J. **1**, 29 (1985); **1**, 67 (1985)

9. K.G. Klimenko, A.S. Vshivtsev, B.V. Magnitsky, V.Ch. Zhukovsky, Fiz. Elem. Chast. Atom. Yadra **29**, 1259 (1998); V.V. Vladimirsky, Yad. Fiz. **58**, 107 (1995); **62**, 2240 (1999); **59**, 2063 (1996); V.V. Vladimirsky, D.V. Peregoudov, Yad. Fiz. **63**, 1534 (2000)
10. V.Ch. Zhukovskii, K.G. Levtchenko, T.L. Shoniya, P.A. Eminov, Moscow University Physics Bulletin, **51**, No. 1, 1 (1996), hep-ph/9810244; V.Ch. Zhukovskii, O.F. Dorofeev, A.V. Borisov, Synchrotron Radiation in a Superstrong Magnetic Field, in Synchrotron Radiation Theory and its Development (Series on Synchrotron Radiation Technique and Applications, Vol. 5), edited by V.A. Bordovitsyn (World Scientific, Singapore 1998), pp. 350–400; I.M. Ternov, V.G. Bagrov, V.A. Bordovitsin, O.F. Dorofeev, Zh. Eksp. Teor. Fiz. (Sov. Phys. JETP) **55**, 2273 (1968)
11. Sh.S. Agaev, A.S. Vshivtsev, V.Ch. Zhukovsky, P.G. Midodashvili, Vestn. Mosk. Univ. Fiz. **26**, 12 (1985)
12. A.S. Vshivtsev, V. Ch. Zhukovskii, P.G. Midodashvili, A.V. Tatarintsev, Izv. Vissh. Ucheb. Zav., Fizika 5 (Sov. Phys. J. **N5**, 47 (1986))
13. F. Schwabl, Quantum mechanics (Springer-Verlag 1995); S. Flugge, Practical quantum mechanics (Springer-Verlag 1994); A.A. Sokolov, I.M. Ternov, V. Ch. Zhukovskii, Kvantovaya Mechanica (M. Nauka 1979)